**Significant Digits Reference Sheet**

Every experiment (except in some counting situations) involves a degree of uncertainty.

Suppose that Jasmine, Matt and Christophe measure the length of a sheet of paper, using a ruler that is divided into tenths of a centimetre (illustrated in figure 1.1) and record the results in Table 1: Paper Width

**Figure 1.1: Measuring Paper**

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

**Table 1: Paper Width**

|  |  |
| --- | --- |
| Team Member | Paper Width (cm) |
| Jasmine | 8.65 |
| Christophe | 8.68 |
| Matt | 8.62 |

Notice that everyone agrees as to the first two digits. Clearly the third digit, which has been estimated by everyone, is an uncertain figure.

The digits that are considered certain and the first estimated digit are called significant figures.The number of significant figures in a measurement depends upon the precision of the instrument. By applying the rules of significant figures, the calculated quantities will carry the same accuracy as the initial measured quantities.

**What digits are significant?**

**Rule #1:** All non-zero digits are always considered significant. So 24 has two significant digits, and 24.1 has three significant digits.

**Rule #2:** All zeros bounded by non-zero integers are always considered significant. So 2004 has four significant digits.

**Rule #3:** Zeroes placed before other digits (called leading zeros) are not significant; 0.024 has two significant digits.

**Rule #4 (a):** Zeroes at the end of a number are significant ONLY if they come after a decimal point; 2.40 has three significant digits.

**Rule #4 (b):** (*The ambiguous zero*) It is impossible to tell if zeros at the end of a number with no decimal point are significant. For example, in the number 8200, it is not clear if the zeroes are significant or not. The number of significant digits in 2400 is at least two, but could be three or four. To avoid uncertainty, we use scientific notation to place significant zeroes behind a decimal point:

2.400 X 103 has four significant digits

2.40 X 103 has three significant digits

2.4 X 103 has two significant digits

A number is often expressed in scientific notation to make the number of significant figures more apparent. By convention, *when a number is expressed in scientific* notation, any type of zero (either leading or trailing zeros) is considered to be significant**.**

**Significant figures and operations**

1. **Addition/Subtraction:**

For addition and subtraction, the answer must be rounded to the **same**

**decimal place** (not significant digits)as theleast number of decimal places in any of the numbers being added or subtracted. This can changethe number of significant figures in the course of a multi-step calculation.

Example:

2.42 J (two decimal places)   
4.2 J (one decimal place)

+ 0.6642 J (four decimal place)   
 7.3 J (one decimal place)

1. **Multiplication/Divisions/Trigonometric Functions:**

In a calculation involving multiplication, division and trigonometric functions the number of significant digits in the answer should be equal to the least number of significant digits in any one of the numbers being multiplied, divided etc.

**Example #1:**

If an appliance uses 7.3 J for 1264 seconds then the power consumed by the appliance is (P = E • t):

7.3 J (two significant digits)

X 1264 sec (four significant digits)   
 9.2 x 103 W (two significant digits)

**Example #2:**

Φ

6.9 cm

14.7 cm

What is the value of angleΦ?

To solve for angleΦ we use the cosine function.

In this example the adjacent side measures 6.9 cm (2 significant digits) and the hypotenuse measures 14.7 (three significant digits), therefore the answer should have two significant digits.

**Example #3:**

Using the quadratic formula to solve for the zeros of the function:

In this example a= -10 (limited to one significant digit because of the ambiguous zero) b= -5 (one significant digit) c = 11 (two significant digits) therefore:

|  |  |
| --- | --- |
|  | **Step 1:** The multiplication of -4ac and the denominator are limited to one significant digit by the -10. The square of -5 is limited to one significant digit.*(By rules of intermediate answers – see below – keep one extra significant digit in intermediate answers.)* |
|  | **Step 2:** The addition under the radical keeps the number of decimal places but the step before limits the 465 to two digits |
|  | **Step 3:** The radical is limited two digits (see step 1) |
| OR | **Step 4:** The addition/subraction keeps the number of decimal places |
| OR | **Step 5:** The division is limited to one significant digit by the -20 as well as the previous steps. |

1. **Logarithms & Antilogarithms:**

|  |
| --- |
| * The logarithm (base 10) of x, log x = a, where x = 10a. |
| * The antilogarithm (base 10) of a, antilog a = x, where x = 10a. |
| * A logarithm is divided into two (2) parts by the decimal:   + The **characteristic** is the power of 10 when a number is written in scientific notation.   + The **mantissa** is the numbers after the decimal when written in scientific notation   + The mantissa has as many significant figures as the number whose log was found.   + For the inverse log (antilogarithm) the answer will have as many significant figures as the number of digits in the mantissa.   **For example**:  *Number of significant figures*  *Characteristic*  *Mantissa*  Since 5.27 X 103 has three significant figures then the mantissa will have 3 decimal places. |
| **Example 2:** What is thepH of a solution with the [H+] = 3.44 X 10-4 M.  Because 3.44 has 3 significant digits, we round the mantissa to three decimal places; therefore the pH of the solution is 3.463. |
| **Example 3:** What is the [H+] in a solution with a pH of 4.55?  Because the mantissa has 2 digits the pH can have only 2 significant figures, therefore the pH of the solution is 2.8 X 10-5 M. | | |

**VI. Intermediate Answers**

When doing multi-step calculations, **keep at least one more significant digit in intermediate results** than needed in your final answer.

For instance, if a final answer requires two significant digits, then carry at least three significant digits in calculations. If you round-off all your intermediate answers to only two digits, you are discarding the information contained in the third digit, and as a result the *second* digit in your final answer might be incorrect. (This phenomenon is known as "round-off error.")

1. **Rules for rounding**

When the answer to a calculation contains too many significant figures, it must be rounded off.

There are 10 digits that can occur in the last decimal place in a calculation. One way of rounding off involves underestimating the answer for five of these digits (0, 1, 2, 3, and 4) and overestimating the answer for the other five (5, 6, 7, 8, and 9). This approach to rounding off is summarized as follows.

* If the digit is smaller than 5, drop this digit and leave the remaining number unchanged. Thus, 1.684 becomes 1.68.
* If the digit is 5 or larger, drop this digit and add 1 to the preceding digit. Thus, 1.247 becomes 1.25.